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NON-LINEAR RENEWAL THEORY FOR LATTICE RANDOM WALKS.(U)

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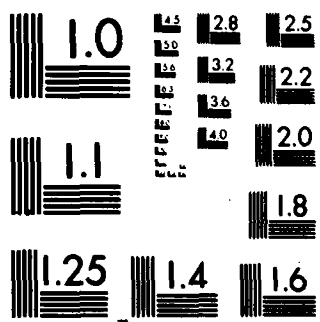

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BY

STEVE LALLEY

TECHNICAL REPORT NO. 9

JUNE 1, 1980

PREPARED UNDER CONTRACT
N00014-77-C-0306 (NR-042-373)
OFFICE OF NAVAL RESEARCH

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1. Introduction

Let $\{X_n\}_{n\geq 1}$ be iid with mean $\mu > 0$ and variance $\sigma^2 < \infty$, $S_n = \Sigma_{j=1}^n X_j$, and $Z_n = S_n + \xi_n$, where ξ_n is, for each n, independent of the sequence $X_{n+1}, X_{n+2}, \ldots$. Under various assumptions concerning the nature of the process $\{\xi_n\}$, Lai and Siegmund ([1], [2]) developed a "renewal theory" for $\{Z_n\}$ and demonstrated its usefulness in sequential statistical analysis. Their results, however, were derived under the standing assumption that the random walk $\{S_n\}$ be nonlattice; this is sometimes troublesome in statistical problems where discrete data is involved.

The purpose of this note is to state the appropriate analogues of the Lai-Siegmund results for the case of a lattice walk $\{S_n\}_{n\geq 1}$ and indicate briefly how their proofs should be modified. A special case of one of these results (Theorem 1) was recently obtained by Hagwood and Woodroofe ([1]) via a rather different approach; however, for the purposes of sequential statistics, the more useful result would seem to be Theorem 3, which gives precise information concerning the hitting times involved.

Acknowledgement. The author has had illuminating conversations with D. Siegmund and M. Woodroofe on the subject of renewal theory.

2. Statement of Results

We will assume throughout that the walk $\{S_n\}$ is supported by the lattice $h \cdot \mathbb{Z}$, and also that

(1) $$\xi_n \xrightarrow{\mathcal{D}} \xi$$

where the limit random variable ξ has a continuous distribution function. (It is easy to find counterexamples to Theorems 1 and 3 when the distribution of ξ has discontinuities: *for example*, when ξ_n oscillates between $+\epsilon_n$ and $-\epsilon_n$ for some sequence $\epsilon_n \downarrow 0$.) In addition we will use the following notations and conventions:

(2) $$T = T_a = \min\{n : Z_n > a\}$$

$$\tau_+ = \min\{n : S_n > 0\}$$

$$g(k) = [ES_{\tau_+}]^{-1} P\{S_{\tau_+} \geq hk\} \quad ; \qquad k = 1,2,\ldots$$

$$G(x) = \Sigma_{k=1}^{[\![x/h]\!]} g(k) \quad ; \qquad x > 0$$

$$G_y(x) = G(x+y) \quad ; \qquad 0 < y \leq h$$

$$H_y(x) = \Sigma_{k\in\mathbb{Z}} P\{kh + y < \xi \leq kh + y + x\} \quad ; \qquad 0 < x,y \leq h$$

THEOREM 1: *Assume that for some* δ, $1/2 < \delta < 1$,

(3) $$a^{-\delta}(T_a - \mu^{-1} a) \xrightarrow{P} 0$$

and that for each $\eta > 0$ *there exists* $\rho > 0$ *for which*

(4) $$P\{\max_{n\le j\le n+2\rho n^{\delta}} |\xi_j - \xi_n| > \eta\} \to 0 \quad .$$

Then for all x,y,t such that $0 < x,y \le h$ and $t \in R$, and each $k \in \{0,1,2,\ldots\}$,

(5) $$P\{kh < Z_{T_a} - a \le kh + x\} \to H_y(x)\cdot g(k+1)$$

as $a \to \infty$ through the coset $y + h\mathbb{Z}$. Furthermore, as $a \to \infty$ through R,

(6) $$P\{T_a \le a\mu^{-1} + t\sigma\mu^{-3/2}\ a^{1/2}\} \to \Phi(t) \quad .$$

(Φ is the standardized Gaussian distribution function.) If in addition to the previous assumptions

(7) $$P\{\xi_n \le y\ ;\ (S_n - n\mu)/n^{1/2}\ \sigma \le \omega\} \to \Psi(y,\omega)$$

for some two-dimensional distribuion function Ψ, then

(8) $$P\{kh < Z_{T_a} - a \le kh + x\ ;\ \ T_a \le a\mu^{-1} + t\sigma\mu^{-3/2}\ a^{1/2}\}$$

$$\to g(k+1) \sum_{j\in\mathbb{Z}} \int_{\{j+y<\xi\le j+y+x;\ \zeta>-t\}} d\Psi(\xi,\zeta)$$

as $a \to \infty$ through $y + h\mathbb{Z}$.

It is worth noting that, in contrast to the nonlattice case, the extra condition (7) is essential to the joint convergence of $(Z_{T_a} - a)$ and $(T_a - a\mu^{-1})/a^{1/2}$. Moreover, $(Z_{T_a} - a)$ and $(T_a - a\mu^{-1})/a^{1/2}$ will in general be asymptotically independent *only* when ξ_n and $(S_n - n\mu)/n^{1/2}\ \sigma$ are asymptotically independent: that is, if $\Psi(y,\omega) = P\{\xi \le y\}\cdot\Phi(t)$, then

(9) $$P\{kh < Z_{T_a} - a \le kh + x \; ; \; T_a \le a\mu^{-1} + t\sigma\mu^{-3/2} a^{1/2}\}$$

$$\to g(k+1)H_y(x)\Phi(t) \quad .$$

In many applications, ξ_n and $(S_n - n\mu)/n^{1/2}\sigma$ will be highly dependent (in fact, ξ_n is often a constant multiple of $(S_n - n\mu)^2/n$) and so a limit distribution somewhat different from that given in (9) will occur.

The analogue of Blackwell's Theorem for the process $\{Z_n\}$ is amusing in that the limit is <u>not</u> a Haar measure.

THEOREM 2: Suppose in addition to (1) that there exists δ, 1/2 $1/2 < \delta \le 1$, such that the following three conditions hold:

(10) $$E|X_1|^{2/\delta} < \infty$$

(11) for each $\epsilon > 0$ $$\Sigma_1^\infty P\{|\xi_n| > n^\delta \epsilon\} < \infty$$

and for each $\epsilon > 0$ there exists $\rho > 0$ such that

(12) $$\sum_{j=n}^{n+\rho n^\delta} P\{|\xi_j - \xi_n| \ge \epsilon\} \to 0 \quad \text{as} \quad n \to \infty \quad .$$

Then for each x,y such that $0 < x,y \le h$,

(13) $$\Sigma_1^\infty P\{a < Z_n \le a + x\} \to H_y(x)/\mu$$

as $a \to \infty$ through the coset $y + h\mathbb{Z}$.

Next we give an asymptotic expansion of ET_a.

THEOREM 3: Assume for some $\eta > 0$

(14) $$P\{T \leq \eta a\} = o(a^{-1}), \quad a \to \infty \quad ,$$

and also that the sequence $\{\xi_n\}$ satisfies

(15) $$\Sigma_n P\{\sup_{k \geq n} k^{-\delta}|\xi_k| > \epsilon\} < \infty \quad , \quad \epsilon > 0$$

(16) $$\Sigma_{n \leq j \leq n+n^{\delta}} P\{|\xi_j - \xi_n| \geq \epsilon\} \to 0 \quad \text{as} \quad n \to \infty$$

(17) $$\{\max_{n \leq j \leq n+n^{\delta}} |\xi_j|\}_{n \geq 1} \quad \text{is uniformly integrable} \quad .$$

Then as $a \to \infty$ through $y + h\mathbb{Z}$, $0 < y \leq h$,

(18) $$\mu ET_a = a - E\xi + (ES^2_{\tau_+}/2ES_{\tau_+}) - h$$
$$+ \int_0^h x\, H_y(dx) + o(1) \quad .$$

Random processes of the form $Z_n = S_n + \xi_n$ and stopping rules of the genre $T = \min\{n : Z_n > a\}$ occur frequently in sequential statistical analysis. An important class of such processes is given by

(19) $$Z_n = ng(\overline{Y}_n)$$

where $\overline{Y}_n = (Y_1 + \ldots + Y_n)/n$; $Y_1, \ldots, Y_n, \ldots$ is an iid sequence of p-dimensional random vectors; and $g : R^p \to R^1$ is a C^2 function with the property $g(EY_1) > 0$. Expanding g in a 2-term Taylor series about EY_1, we find that $Z_n = S_n + \xi_n$ for a random walk S_n drifting to $+\infty$ and a sequence ξ_n converging in law to a continuous distribution

(namely a weighted sum of independent χ_1^2 variables).

EXAMPLE: Let $Y_1,\ldots,Y_n,\ldots$ be iid with $P\{Y_j = 1\} = p = 1 - P\{Y_1 = 0\}$ for $0 < p < 1$. In order to estimate $\log(p/q)$ by an estimator with preassigned variance a^{-1}, Robbins and Siegmund [4] defined the stopping rule

$$T_a = \min\{n : \beta_n \cdot (n - \beta_n) > na\} \tag{20}$$

where $\beta_n = Y_1 + \ldots + Y_n$, and proposed the estimator

$$\log[(\beta_{T_a} + 1/2)/(T_a - \beta_{T_a} + 1/2)] \quad . \tag{21}$$

They showed that as $a \to \infty$ this estimator is asymptotically normal with mean $\log\,(p/(1-p))$ and variance $\sim 1/a$ regardless of p, and that $E_pT_a \sim a/p(1-p)$. Subsequently, Siegmund [6] noted that as a consequence of the nonlinear renewal theory for nonlattice walks,

$$p(1-p)E_pT_a = a + (2p-1)^2/2 + p(1-p)/2 + o(1) \tag{22}$$

as $a \to \infty$, provided $[p/(1-p)]^2$ is irrational. It now follows from our Theorem 3 that if $[p/(1-p)]^2$ is rational, $[p/(1-p)]^2 = r/h$ in lowest terms, then

$$\begin{aligned} p(1-p)E_pT_a &= a + (1-2p)^2/2 + p(1-p)/2 \\ &\quad - \int_{u=0}^{q^2h^{-1}} \sum_{k\in\mathbb{Z}} P\{kq^2h^{-1} + y < \xi \le kq^2h^{-1} + y + u\}du + o(1) \end{aligned} \tag{23}$$

as $a \to \infty$ through $y + h\mathbb{Z}$; here $-\xi \sim \chi_1^2$.

3. Proof of Theorem 1

We will present a complete proof only for Theorem 1. Theorem 3 follows from Theorem 1 via Wald's Identity:

$$\begin{aligned}\mu E T_a &= E S_{T_a} \\ &= E Z_{T_a} - E \xi_{T_a} \\ &= a + E(Z_{T_a} - a) - E\xi_{T_a} \quad .\end{aligned}$$

The assumption $\xi_n \xrightarrow{\mathcal{D}} \xi$ suggests $E\xi_{T_a} \to E\xi$, and Theorem 1 leads one to hope that

$$\lim_{\substack{a\to\infty \\ a\varepsilon y+h\mathbb{Z}}} E(Z_{T_a} - a)$$

exists and is the mean of the limiting distribution recorded in (5). The details of this argument are so similar to those given in Theorem 3 of Lai and Siegmund [3] that we omit them.

Theorem 2 requires considerably more care. However, a proof can be distilled from the ideas contained in the proof of Theorem 1 and in the papers of Lai and Siegmund, so we refrain from presenting it.

We will assume for the proof of Theorem 1 that the span h of the lattice supporting the random walk $\{S_n\}$ is 1. Now the assumption $a^{-\delta}(T_a - a\mu^{-1}) \xrightarrow{P} 0$ implies that there is a function $\rho(a) \downarrow 0$ as $a \to \infty$ such that

(24) $$P\{a^{-\delta}|T_a - a\mu^{-1}| \geq \rho(a)\} \to 0 \quad .$$

We are certainly free to let $\rho(a) \downarrow 0$ as slowly as we like; thus we will assume

(25) $$\rho(a)a^{\delta-\frac{1}{2}} \to \infty \quad .$$

Define

(26) $$n_0 = n_0(a) = [\![a\mu^{-1} - \rho(a)a^{\delta}]\!]$$

$$\tau_a = \min\{n > n_0(a) : S_n + \xi_{n_0} > a\}$$

LEMMA 1: Under the conditions (1), (3), and (4),

(27) $$P\{T_a \neq \tau_a\} \to 0$$

and for every $\eta > 0$

(28) $$P\{|Z_{\tau_a} - S_{\tau_a} - \xi_{n_0(a)}| > \eta\} \to 0 \quad .$$

PROOF: Lemma 3 below guarantees that $P\{\tau_a \notin [n_0, n_0 + 2\rho(a)a^{\delta}]\} \to 0$. On the event $\tau_a \in [n_0, n_0 + 2\rho(a)a^{\delta}]$,

$$|Z_{\tau_a} - S_{\tau_a} - \xi_{n_0(a)}| = |\xi_{\tau_a} - \xi_{n_0(a)}|$$

$$\leq \max_{n_0 \leq j \leq n_0 + 2\rho(a)a^{\delta}} |\xi_j - \xi_{n_0}|$$

and since $\rho(a) \downarrow 0$, assumption (4) implies

$$P\{\max_{n_0 \leq j \leq n_0 + 2\rho(a)a^{\delta}} |\xi_j - \xi_{n_0}| > \eta\} \to 0 \quad .$$

This proves (28).

Next, fix $\eta > 0$; then

$$\{T_a \neq \tau_a\} \subset \{T_a \notin [n_0, n_0 + 2\rho(a)a^\delta]\}$$

$$\cup \{\tau_a \notin [n_0, n_0 + 2\rho(a)a^\delta]\}$$

$$\cup \{\max_{n_0 \le j \le n_0 + 2\rho(a)a^\delta} |\xi_j - \xi_{n_0}| > \eta\}$$

$$\cup \{\exists\, n \in [n_0, n_0 + 2\rho(a)a^\delta] :$$

$$S_n + \xi_{n_0} \in [a - 2\eta, a + 2\eta]\} \quad .$$

By (24) $P\{T_a \notin [n_0, n_0 + 2\rho(a)a^\delta]\} \to 0$; by Lemma 3, $P\{\tau_a \notin [n_0, n_0 + 2\rho(a)a^\delta]\} \to 0$ and $P\{\exists\, n \in [n_0, n_0 + 2\rho(a)a^\delta] : S_n + \xi_{n_0} \in [a - 2\eta, a + 2\eta]\}$ is small if a is large and η small; and by assumption (4), $P\{\max_{n_0 \le j \le n_0 + 2\rho(a)a^\delta} |\xi_j - \xi_{n_0}| > \eta\} \to 0$. This proves (27).

The objective now will be to show that $(S_{\tau_a} + \xi_{n_0} - a)$ has the limiting distribution advertised in (5). Before doing so, we recall a useful result from standard renewal theory.

LEMMA 2: Let $\nu = \nu_a = \min\{n : S_n > a\}$ and let $y \in (0,1]$. Then as $a \to \infty$ through $y + \mathbb{Z}$,

$$P\{S_{\nu_a} - a \le x \, ; \, \nu_a \le a\mu^{-1} + t\sigma\mu^{-3/2} a^{1/2}\} \tag{29}$$

$$\to \Phi(t) G_y(x)$$

for all $x > 0$, and all $t \in R$.

For a proof of this result see Siegmund [5].

LEMMA 3: For all x,y with $0 < x,y \leq 1$, and each $k \in \{0,1,2,\dots\}$, if (1) holds, then

(30) $$P\{S_{\tau_a} + \xi_{n_0(a)} - a \in (k, k+x]\} \to H_y(x)g(k+1)$$

as $a \to \infty$ through $y + \mathbb{Z}$. Moreover,

(31) $$P\{\tau_a \leq a\mu^{-1} + t\sigma\mu^{-3/2} a^{1/2}\} \to \Phi(t) \quad \text{as} \quad a \to \infty \quad ,$$

and for all $\epsilon > 0$ there is an $A = A(\epsilon)$ and $\eta = \eta(\epsilon) > 0$ such that $a > A$ implies

(32) $$P\{S_n + \xi_{n_0} \in [a-\eta, a+\eta] \quad \text{for some} \quad n > n_0(a)\} \leq \epsilon \quad .$$

Finally, if (7) holds, then for $x,y \in (0,1]$ and $k \in \{0,1,2,\dots\}$,

(33) $$P\{k < S_{\tau_a} + \xi_{n_0} - a \leq k+x \,;\, \tau_a \leq a\mu^{-1} + t\sigma\mu^{-3/2} a^{1/2}\}$$
$$\to g(k+1) \sum_{j \in \mathbb{Z}} \int_{\{j+y \leq \xi \leq j+y+x;\ \zeta > -t\}} d\Psi(\xi,\zeta)$$

as $a \to \infty$ through $y + \mathbb{Z}$.

PROOF: It is evident that $a - S_{n_0(a)} - \xi_{n_0(a)} \xrightarrow{P} +\infty$ since $[a\mu^{-1} - n_0(a)]a^{-1/2} \to +\infty$. Thus we may use the result of Lemma 2 for the random walk $0, S_{n_0(a)+1} - S_{n_0(a)}, S_{n_0(a)+2} - S_{n_0(a)}, \dots$ and the hitting time $\nu_{a-S_{n_0}-\xi_{n_0}}$ to estimate $P\{S_{\tau_a} + \xi_{n_0} - a \leq z | \mathcal{F}_{n_0}\}$:

(34) $$P\{|P\{S_{\tau_a}+\xi_{n_0}-a\leq z|\mathfrak{F}_{n_0}\}-G_{(y-\xi_{n_0})^*}(z)|>\epsilon\}\to 0$$

as $a\to\infty$ through $y+\mathbb{Z}$,

for every $\epsilon>0$, where

(35) $$u^*=u \bmod 1 \quad \text{for all} \quad u\in\mathbb{R} \ .$$

Since $\xi_{n_0}\xrightarrow{\mathcal{D}}\xi$, and ξ has a continuous distribution,

(36) $$EG_{(y-\xi^*_{n_0})}(z)\to EG_{(y-\xi)^*}(z) \quad ;$$

this and (34) prove (30) since

(37) $$EG_{(y-\xi)^*}(z)=E\sum_{j=1}^{[\![(y-\xi)^*+z]\!]}g(j)$$

$$=\sum_{j=1}^{[\![z]\!]}g(j)+g([\![z+1]\!])P\{[\![(y-\xi)^*+z]\!]=[\![z+1]\!]\}$$

$$=\sum_{j=1}^{[\![z]\!]}g(j)+g([\![z+1]\!])H_y(z-[\![z]\!]) \quad .$$

Similarly we may deduce from Lemma 2

(38) $$P\{|P\{S_{\tau_a}+\xi_{n_0}-a\leq z\,;\ \tau_a\leq a\mu^{-1}+t\sigma\mu^{-3/2}a^{1/2}|\mathfrak{F}_{n_0}\}$$

$$-G_{(y-\xi_{n_0})^*}(z)\,1\{(S_{n_0}+\xi_{n_0}-n_0\mu)/\sigma n_0^{1/2}>-t\}|>\epsilon\}\to 0$$

(as $a\to\infty$ through $y+\mathbb{Z}$) .

The reasoning is as follows: the event

(39) $$\{S_{\tau_a} + \xi_{n_0} - a \le z\ ;\ \tau_a \le a\mu^{-1} + t\sigma\mu^{-3/2}\, a^{1/2}\}$$

$$= \{(S_{n_0+\nu} - S_{n_0}) - (a - S_{n_0} - \xi_{n_0}) \le z\ ;$$

$$\nu \le (a\mu^{-1} - n_0) + t\sigma\mu^{-3/2}\, a^{1/2}\}$$

where

(40) $$\nu = \nu_{a-S_{n_0}-\xi_{n_0}} = \min\{k : S_{n_0+k} - S_{n_0} > a - S_{n_0} - \xi_{n_0}\}\quad .$$

By Lemma 2 and the fact that $a - S_{n_0} - \xi_{n_0} \xrightarrow{P} +\infty$

(41) $$P\{(S_{n_0+\nu} - S_{n_0}) - (a - S_{n_0} - \xi_{n_0}) \le z\ ;$$

$$\nu \in (a - S_{n_0} - \xi_{n_0})\mu^{-1} \pm (a - S_{n_0} - \xi_{n_0})^{\delta} | \mathfrak{F}_{n_0}\}$$

$$- G_{(y-\xi_{n_0})^*}(z) \xrightarrow{P} 0\quad ;$$

moreover, since $(a - S_{n_0} - \xi_{n_0})^{\delta}/n_0^{1/2} \xrightarrow{P} 0$, and $a^{1/2}/n_0^{1/2}\,\mu^{1/2} \to 1$,

(42) $$1\{(a - S_{n_0} - \xi_{n_0})\mu^{-1} \pm (a - S_{n_0} - \xi_{n_0})^{\delta}$$

$$\le (a\mu^{-1} - n_0) + t\sigma\mu^{-3/2}\, a^{1/2}\}$$

$$- 1\{(S_{n_0} + \xi_{n_0} - n_0\mu)/\sigma n_0^{1/2} > -t\} \xrightarrow{P} 0\quad .$$

This and (41) imply (38).

If we set $z = +\infty$ in (38), we obtain (31). Furthermore, it is clear that if (7) holds, then (38) implies (33).

To prove (32) we choose η $(0 < \eta < 1/2)$ small enough and A large enough so that $a > A$ implies

$$P\{\xi^{*}_{n_0(a)} \in I_{2\eta}\} < \epsilon \tag{43}$$

for every interval $I_{2\eta} \subset [0,1]$ of length 2η (this is possible since $\xi_n \xrightarrow{\mathcal{D}} \xi$ and ξ has a continuous distribution). Since the random walk $\{S_n\}$ is supported by $\mathbb{Z}$, it is clear that (43) implies (32).

Theorem 1 is an immediate consequence of Lemmas 1 and 3.

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Renewal theorems are obtained for a class of perturbed arithmetic random walks. An application to the study of fixed-width confidence intervals is discussed.

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